GPDs on and off the Lattice

Andreas Schäfer (Regensburg) for RQCD & D. Ivanov, ...

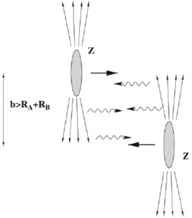
- Exclusive quarkonium production at NLO & resummation
- Momentum smearing in LQCD
- Λ polarization in heavy ion collisions

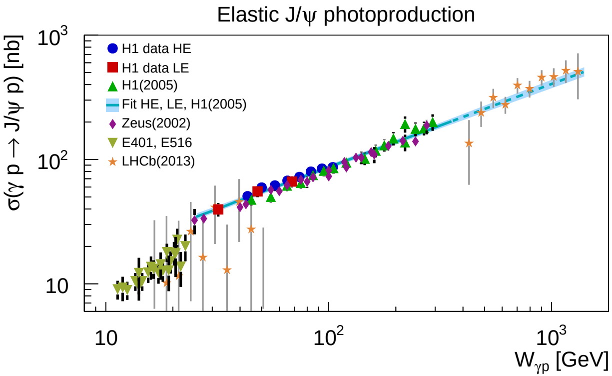
SFB/
TR 55

Exclusive vector meson production

- Exclusive quarkonium photoproduction is a key process to determine the gluon GPDs H^g and E^g at EIC, see e.g. Koempel, Kroll, Metz and Zhou, arXiv:1112.1334.
- We calculated this process at NLO and found huge scale uncertainties. Ivanov, AS, Szymanowski and Krasnikov, hep-ph/0401131
- Now we, Ivanov and (Braun, Manashov, AS) and (Pire Szymanowski, Wagner) try to improve using resummation
- Crucial: The Catani and Hautmann approach for DIS, hep-ph/9405388, can be directly generalized to exclusive reactions

Already now there exists very good data from ultra peripheral collisions





$$\sigma^{AB} = \int dk_A \frac{dn^A}{dk_A} \sigma^{\gamma B}(W_A(k_A)) + \int dk_B \frac{dn^B}{dk_B} \sigma^{\gamma A}(W_B(k_B))$$

where $k_{A,B} = \frac{1}{2} x_{A,B} \sqrt{s}$.

Combined Collinear QCD and NRQCD factorizations:

twist $\sim 1/M^n$ and velocity $\sim \left(\frac{v}{c}\right)^m$ expansions:

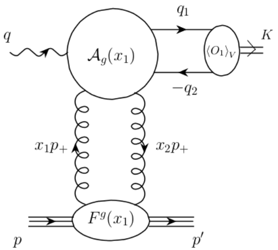


Figure 1: Kinematics of heavy vector meson photoproduction.

$$\mathcal{M} \sim \left(\frac{\langle O_1 \rangle_V}{m^3}\right)^{1/2} \int_{-1}^{1} dx \left[T_g(x,\xi)\, F^g(x,\xi,t) + T_q(x,\xi) F^{q,S}(x,\xi,t) \right]$$

$$F^{q,S}(x,\xi,t) = \sum_{q=u,d,s} F^q(x,\xi,t)$$

$F^{g(q)}(x, \xi, t; \mu_F^2)$ – the gluon (quark) GPDs, m is a pole mass of heavy quark,

$\xi = M^2/(2W^2 - M^2)$ is the skewedness parameter.

In NRQCD all information about the quarkonium structure is encoded in the NRQCD matrix element $\langle O_1 \rangle_V$ which enters the leptonic decay rate

$$\Gamma[V \to l^+ l^-] = \frac{2e_q^2 \pi \alpha^2}{3} \frac{\langle O_1 \rangle_V}{m^2} \left(1 - \frac{8\alpha_S}{3\pi}\right)^2 .$$

The hard scattering kernels:

$$
\begin{aligned}
T_g(x,\xi) &= \frac{\xi}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)}\mathcal{A}_g\left(\frac{x-\xi+i\varepsilon}{2\xi}\right) \\
T_q(x,\xi) &= \mathcal{A}_q\left(\frac{x-\xi+i\varepsilon}{2\xi}\right).
\end{aligned}
$$

LO

$$\mathcal{A}_g^{(0)}(y) = \alpha_S \qquad \mathcal{A}_q^{(0)}(y) = 0$$

NLO

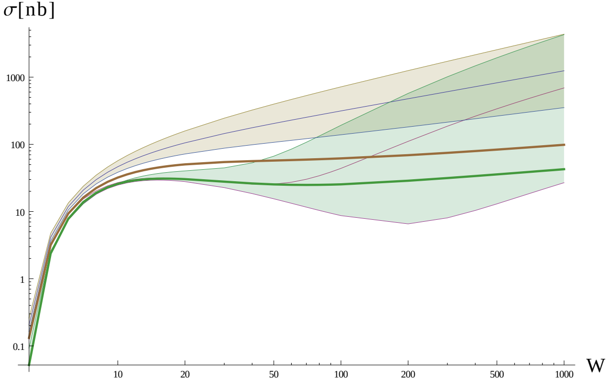
D. Ivanov, AS , L. Szymanowski and G. Krasnikov - (2004);
recently S.Jones, A. Martin, M. Ryskin and T. Teubner - (2016)

$$T_q(x,\xi) = \frac{\alpha_S^2(\mu_R)C_F}{2\pi} f_q\left(\frac{x-\xi+i\varepsilon}{2\xi}\right)$$

$$
\begin{aligned}
f_q(y) &= \ln\left(\frac{4m^2}{\mu_F^2}\right)(1+2y)\left(\frac{\ln(-y)}{1+y}-\frac{\ln(1+y)}{y}\right) \\
&- \pi^2\frac{13(1+2y)}{48y(1+y)}+\frac{2\ln 2}{1+2y} \\
&+ \frac{\ln(-y)+\ln(1+y)}{1+2y}+(1+2y)\left(\frac{\ln^2(-y)}{1+y}-\frac{\ln^2(1+y)}{y}\right) \\
&+ \frac{3-4y+16y(1+y)}{4y(1+y)}Li_2(1+2y) \\
&- \frac{7+4y+16y(1+y)}{4y(1+y)}Li_2(-1-2y)
\end{aligned}
$$

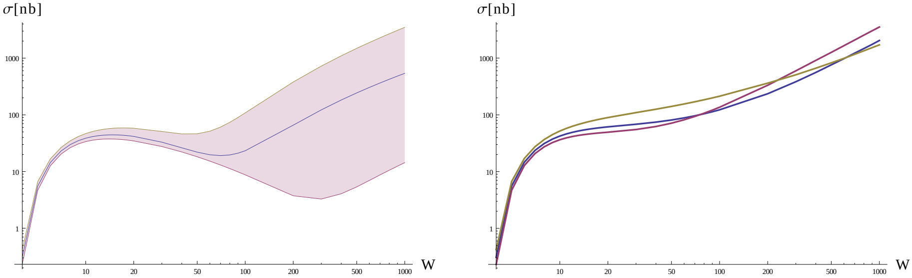
The expression for T_g is much longer

The scale dependence for both LO and NLO is huge, making this reaction channel basically useless for GPD determinations.



Photoproduction cross section as a function of $W = \sqrt{s_{\gamma p}}$ for $\mu_F^2 = M_{J/\psi}^2 \times \{0.5, 1, 2\}$ (bottom to top) in LO (larger) and NLO (smaller). Thick lines for LO and NLO for $\mu_F^2 = 1/4 M_{J/\psi}^2$. Note logarithmic scale

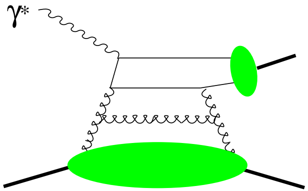
So you have to work still harder: High-energy resummation

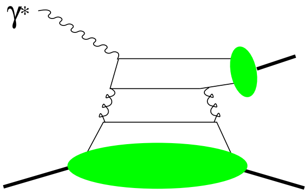


NLO(left panel) and resummed (right panel) photoproduction cross section (only gluonic GPDs included in both cases) as function of $W = \sqrt{s_{\gamma p}}$ for $\mu_F^2 = M^2_{J/\psi} \times \{0.5, 1, 2\}$ (pink, blue and yellow lines respectively) Ivanov, Pire, Szymanowski, Wagner preliminary !!!

What is done ?

- Why NLO corrections are large at small x?
  large contribution comes from $\xi \ll x \ll 1$ ($Q = M$)





$$ImA^g \sim H^g(\xi,\xi) + \frac{N_c\alpha_s}{\pi}\left[\log\frac{Q^2}{\mu_F^2} - \log 4\right]\int\limits_{\xi}^{1}\frac{dx}{x}H^g(x,\xi)$$

$H^g(x,\xi) \sim xg(x) \sim const$ thus $\int dx/xH^g(x,\xi) \sim \log(1/\xi)H^g(\xi,\xi)$

- large NLO correction – first BFKL log of energy
- use BFKL as a tool to improve collinear approach

- The BFKL trick (LLA); Nth moment:

$$\frac{1}{x}\frac{\bar{\alpha}_s^n}{(n-1)!}\log^{n-1}\frac{x}{\xi}\to\left(\frac{\bar{\alpha}_s}{N}\right)^n$$

  High energy resummation is obtained by taking the limit $N\to 0$

- The anomalous dimension after expanding in $\bar{\alpha}/N$ is known

$$\gamma_N=\gamma_{gg,N}=\frac{\bar{\alpha}_s}{N}+2\zeta(3)\left(\frac{\bar{\alpha}_s}{N}\right)^4+2\zeta(5)\left(\frac{\bar{\alpha}_s}{N}\right)^6+\dots$$

- Using Catani's and Hautmann's $\overline{MS}$ result

$$C_N^g\sim h_V(\gamma_N)R_N\left(\frac{Q^2}{\mu_F^2}\right)^{\gamma_N}$$

  with process independent

$$R_N(\alpha_s)=1+\frac{8\zeta(3)}{3}\left(\frac{\bar{\alpha}_s}{N}\right)^3-\frac{\pi^4}{120}\left(\frac{\bar{\alpha}_s}{N}\right)^4+\frac{22\zeta(5)}{5}\left(\frac{\bar{\alpha}_s}{N}\right)^5+\dots$$

- Choosing the process dependent

$$h_V^{J/\psi}(k_\perp^2) = \frac{Q^2}{Q^2 + 4k_\perp^2} \quad \Rightarrow \quad h_V^{J/\psi}(\gamma_N) = 4^{-\gamma_N}\,\Gamma(1+\gamma_N)\,\Gamma(1-\gamma_N)$$

- Result for $\mu_F = M_{J/\psi}$

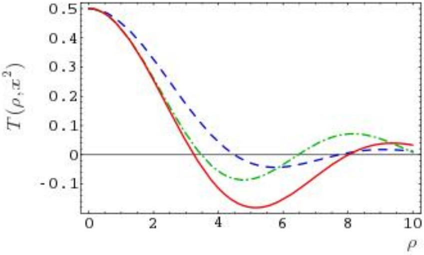
$$C^g_{J/\psi,N} = \alpha_s \left\{ 1 - \log(4)\left(\frac{\bar{\alpha}_s}{N}\right) + \left(\frac{\pi^2}{6} + 2\log^2(2)\right)\left(\frac{\bar{\alpha}_s}{N}\right)^2 + \dots \right\}$$

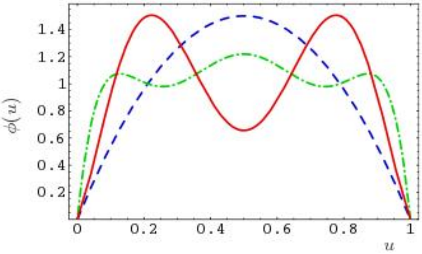
Momentum smearing in LQCD

What is needed to make “Ji’s method” to work quantitatively?
We use the variant V. Braun and D. Müller, EPJ **C 55** (2008) 349, arXiv:0709.1348 where, correlators non-local in space are used to analyze, e.g., the pion distribution amplitude

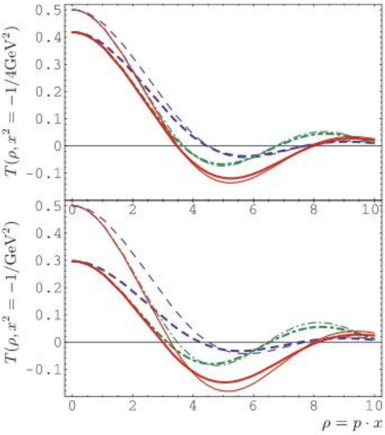
$$\begin{aligned}
\langle 0|T\{j_\mu(x) j_\nu(-x)|\pi^0(p)\rangle &= -\frac{5\mathrm{i}}{p} f_\pi \epsilon_{\mu\nu\rho\sigma} \frac{x^\rho p^\sigma}{8\pi^2 x^4}\; T(p\cdot x, x^2) \\
T(p\cdot x, x^2) &= \int_0^1 du \quad e^{\mathrm{i}(2u-1)p\cdot x} \quad H(u,(\mu x)^2,\alpha_s(\mu))\, \Phi_\pi(u,\mu) \\
&+ \quad \text{higher twist}
\end{aligned}$$

It is completely sufficient to study $x^0 = 0$, $\vec{x} \neq 0$ **IF** $|\vec{x}|$ is small enough for pQCD to apply.





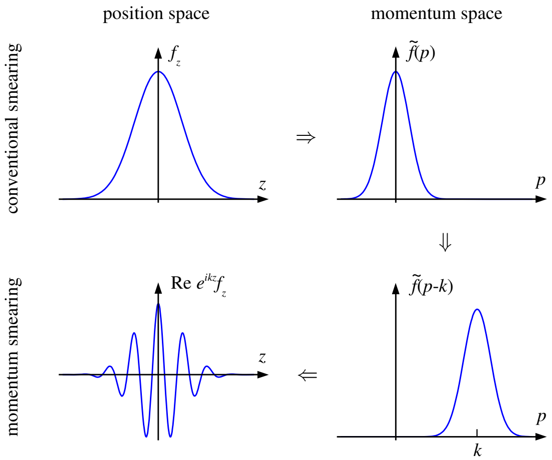
Three model DAs for the pion

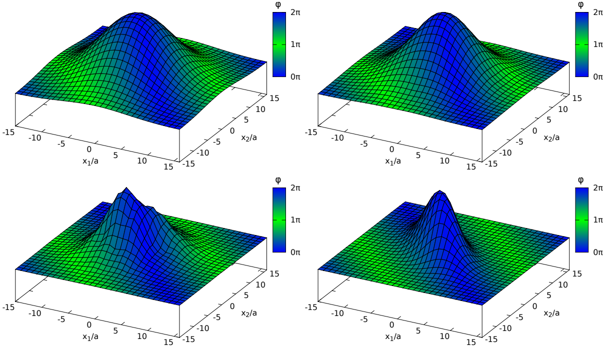


thick lines: NNLO plus twist-four predictions; thin lines LO predictions; $\mu^2 = -1/(\vec{x})^2 = 4\mathrm{GeV}^2$ (upper) and $1\mathrm{GeV}^2$ (lower). The colors correspond to the different DA models.

For small $\vec{x}$ one needs very large momenta

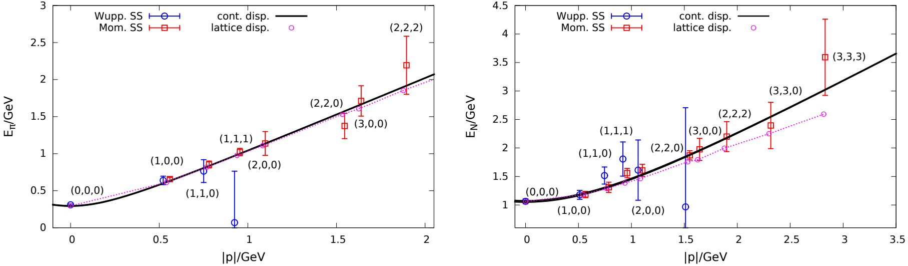
momentum smearing: B. Musch, G. Bali, B. Lang, AS, 1602.05525





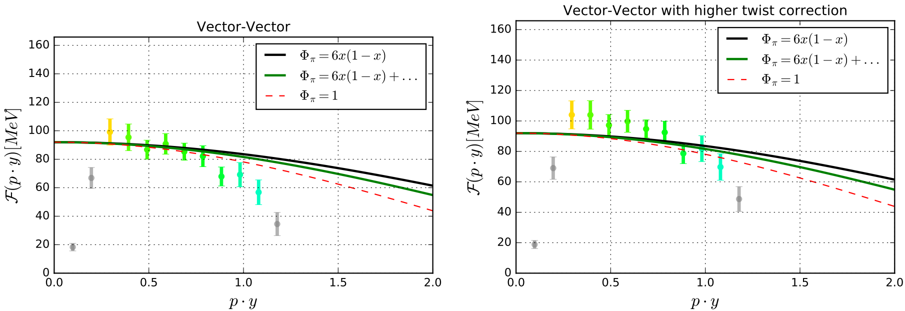
Smearing density profile in the x_1-x_2 plane for $\vec{k} = (1, 1, 0)$. Top left: free field case. Top right: APE smeared gauge links. Bottom left: original gauge links. Bottom right: APE smeared links with an additional boost factor $\gamma = 5.3$.

- dispersion relation



pion nucleon

- Lorentz contraction does not help

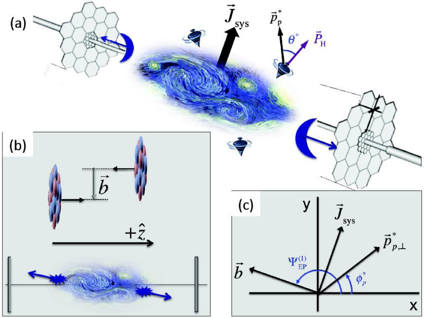


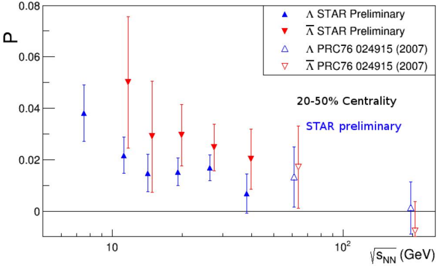
renormalized results for T from the pion DA without (left) with (right) higher twist corrections ($N_f = 2$; 800 configs $\to$ 2000 configs; $m_\pi = 290$ MeV, $a = 0.071$ fm; $32^3 \times 64$; plus higher radiative corrections)

Works as advocated, but you need still larger momenta, i.e. finer lattices (CLS open boundary conditions)

On the look-out for links between hadron structure and heavy ion physics: A very speculative idea

Λ and $\bar{\Lambda}$ produced at mid-rapidity in Heavy Ion Collisions are preferentially polarized along $\vec{J}_{\rm sys}$





STAR; 2016; 32th Winter Workshop on Nuclear Dynamics

How can this happen if the transverse correlation length is $1/Q_s \sim 0.2$ fm ?

Some theoreticians see no problem, e.g. Pang, Petersen, Wang Wang, 1605.04024 “vortical fluid”;
relation to Chiral Vortical Effect

My question: Can there be a significant spin, rapidity correlation in the initial state due to Boer-Mulders correlation?

More general question: Is a fast moving nucleon a coherent quantum state?

Transverse quark density, expressed in terms of GPDs

$$\frac{1}{(2\pi)^2}\int d^2\Delta_\perp e^{ib_\perp\cdot\Delta_\perp}\int\frac{dz^-}{2\pi}e^{ix\bar{P}^+z^-}\langle P_2|\,\bar{q}(-\frac{1}{2}z)\,\gamma^+[1+\vec{s}\cdot\vec{\gamma})\gamma_5]q(\frac{1}{2}z)\,|P_1\rangle\Big|_{z^+=0}^{z_\perp=0}$$

$$= \frac{1}{2}\Big[F+s^iF_T^i\Big]$$

$$= \frac{1}{2}\Bigg[H - S^i\epsilon^{ij}b^j\frac{1}{m}E' - s^i\epsilon^{ij}b^j\frac{1}{m}\Big(E_T'+2\tilde{H}_T'\Big)$$

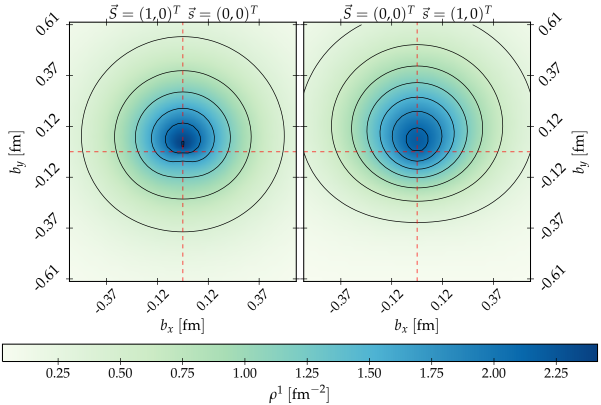
$$+ s^iS^i\Big(H_T-\frac{1}{4m^2}\Delta_b\tilde{H}_T\Big)+s^i(2b^ib^j-b^2\delta^{ij})S^j\frac{1}{m^2}\tilde{H}_T''\Bigg]$$

has a simple interpretation:

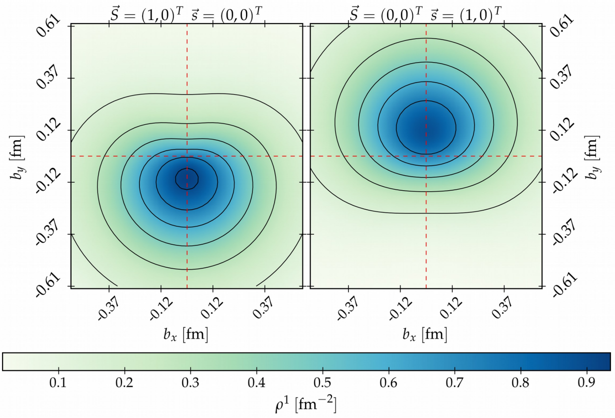
$S^i\epsilon^{ij}b^j$ coupling of proton spin to quark angular momentum

$s^i\epsilon^{ij}b^j$ coupling of quark spin to quark angular momentum

s^iS^i coupling of quark spin and proton spin

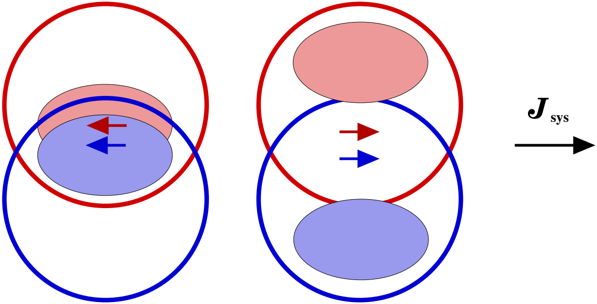


1st x moment of up quarks in a proton, RQCD unpublished



1st x moment of down quarks in a proton, RQCD unpublished

M. Burkardt and B. Hannafious, “Are all Boer-Mulders functions alike?”, arXiv:0705.1573 The longitudinal momentum is increased for $y > 0$ and decreased for $y < 0$



red: proton moving towards the reader; blue: nucleon moving away; quarks polarized to the right have smaller longitudinal momentum and, therefore, are more likely to stay at mid-rapidity.

Needed: Tensor GPDs of nuclei

Conclusions

- High energy (BFKL-) resummation works efficiently for exclusive heavy quark production (as well as DVCS and TCS). Still the theoretical uncertainties are sizable
- Ji's method works, but there is no free lunch: One needs perturbatively small spacial separations and very large momentum. Momentum smearing is efficient to reach this. Everything looks as advertised but one needs very fine lattices and huge statistics to beat the usual moment method.
- A very speculative idea: How tensor GPDs might be related to Λ polarization in HICs